APh 156A: Organization and Policies

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Teaching Assistant: Rory Perkins, 130 Watson, rory@its.caltech.edu

Class list: Please fill out the class list and also fill out your timetable.

Grading policy: Homework 40%, exams 60% (see copy of previous year grading scheme for reference)

Collaboration policy: see attached form

Text: “Fundamentals of Plasma Physics” by P. M. Bellan

Syllabus: The class will be all year (fall, winter, spring quarters) and slightly less than one third of the text will be covered each term so that most, but not all of the text will be covered. The table of contents of the text is given on the following pages. Homework will be assigned as appropriate to the material that has been covered so that the homework load will be variable but significant. A small portion of the homework will involve simple numerical calculations and assistance will be provided for any students unfamiliar with these methods.

Additional Reference books (on reserve in library):

1. Schmidt, The Physics of High Temperature Plasmas
2. Krall and Trivelpiece, Plasma Physics
3. Nicholson, Introduction to Plasma Theory

Prerequisites:

It is assumed that students have some familiarity with electricity and magnetism, Maxwell’s equations, ordinary and partial differential equations, complex analysis, linear algebra, classical mechanics. However, these concepts will be briefly reviewed each time they are first used and students unfamiliar with some of these concepts will be assisted as appropriate.
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1.2 Brief history of plasma physics  
1.3 Plasma parameters  
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Schedule
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