SS/MA 214 "Mathematical Finance"

Course Syllabus – Fall/2023
HSS, California Institute of Technology

Course Instructor: Jaksa Cvitanic, cvitanic@caltech.edu

Office Hours: TBA and by appointment

Class meetings: T Th 5:05-06:30PM

Disclaimer: The syllabus is subject to change.

Course Description

Part 213a is focused on the methods in Mathematical Finance: Pricing financial derivatives, risk management, optimal portfolio selection, and equilibrium in financial markets. Methods of stochastic, Ito calculus for models driven by Brownian motion.

Prerequisites: *A solid knowledge of probability, at a level of an upper level undergrad course. Some exposure to stochastic processes, real analysis/measure theory and partial differential equations is helpful, but not mandatory.*

Course Welcome

There are two main areas we will cover:

- 1. Pricing of options and other financial derivatives
- 2. Optimal portfolio selection

In the first part, we will distinguish between complete markets, in which there will be a unique no-arbitrage price, and incomplete markets, where absence of arbitrage is not sufficient to obtain uniqueness of prices. We will start with discrete-time models, but most of the course will be in the framework of continuous-time, Brownian Motion driven models. An introduction to Stochastic, Ito Calculus will be given. The benchmark model will be the Black-Scholes-Merton pricing model, but we will also cover more general models, such as stochastic volatility models. Models with market frictions such as portfolio constraints will also be considered. We will discuss both the Partial Differential Equations approach, and the Martingale approach. They are related through the notion of Backward Stochastic Differential Equations and the Feynman-Kac theorem.

In the second part we will find optimal portfolio strategies in the above mentioned models, and discuss relationship to pricing and risk management. The two parts are related through the concept of risk-neutral probabilities (or equivalent martingale measures, or state-price densities). We will also study price equilibrium in financial markets.

Learning Outcomes

You will learn the basic methods of mathematical finance. You will NOT learn much about implementation issues, such as statistical estimation of model parameters and numerical computations.

Course Website or Learning Management System

Assessment, Attendance and Participation

40% homeworks, equally weighted, 60% final exam. You must pass the final exam and you must receive 50% of the grade for each homework to pass the course. Penalty for late submission of homework: 10% per day. Penalty for late submission of final: 33% per day.

Academic Integrity

Caltech's Honor Code: "No member of the Caltech community shall take unfair advantage of any other member of the Caltech community."

Understanding and Avoiding Plagiarism: Plagiarism is the appropriation of another person's ideas, processes, results, or words without giving appropriate credit, and it violates the honor code in a fundamental way. You can find more information at: http://writing.caltech.edu/resources/plagiarismLinks to an external site..

Collaboration Policy

Discussions of class material are allowed; fellow students can give hints on homework assignments; no collaboration allowed on the exams. The homeworks and exams are open-book, open-notes. You are not allowed to consult others on the exams. You may not consult any prepared solutions for the homework or exam problems, whether they are from this year or from previous years, from Caltech or external sources, and you must cite any use of material from outside references. All solutions thatare handed in should be written up individually and should re ect your own of the subject matter at the time of writing. Software produced scripts and plots are considered part of your write-up

and should be done individually (you can share ideas, but not code). For group presentations, each individual should contribute approximately the same amount of effort.As a general guideline for the collaboration policy, you should be ableto reproduce any solution you hand in without help from anyone else. It is possible to achieve high scores on the homework, but still fail the exams.This likely indicates poor adherence to the collaboration policy: the object of the HW problems and the collaboration policy is to help you learn the material.

The main textbook:

S. Shreve: Stochastic Calculus for Finance II : Continuous-Time Models

However, we will not always follow the textbook closely. We will use other books, lecture notes and research papers, too, such as:

J. Cvitanic "Theory of portfolio optimization in markets with frictions". (Lecture Notes, that will be provided)

Students who are interested in additional computational, economics and mathematical aspects can also consult the following books:

J. Cvitanic and F. Zapatero: Introduction to the Economics and Mathematics of Financial Markets (introductory level)

K. Back: Asset Pricing and Portfolio Choice Theory (intermediate/advanced)

T. Bjork: Arbitrage Theory in Continuous Time (intermediate/advanced)

B. Dumas and E. Luciano: The Economics of Continuous-Time Finance (advanced)

Moreover, for models with jump processes, one can consult

R. Cont and P. Tankov: Financial Modelling with Jump Processes

Students with Documented Disabilities

Students who may need an academic accommodation based on the impact of a disability must initiate the request with Caltech Accessibility Services for Students (CASS). Professional staff will evaluate the request with required documentation, recommend reasonable accommodations, and prepare an Accommodation Letter for faculty dated in the current quarter in which the request is being made. Students should

contact CASS as soon as possible, since timely notice is needed to coordinate accommodations. http://cass.caltech.edu/ Links to an external site..

Undergraduate students should contact Dr. Lesley Nye, Associate Dean of Undergraduate Students (administrative contact: Beth Larranaga) and graduate students should contact Dr. Kate McAnulty, Associate Dean of Graduate Studies (administrative contact: Jacob Dalton).

Course Schedule (subject to change):

We will mostly follow Shreve's book in the first half of the course, and various material in the second half.

Additional Resources for Students

- **Student Wellness Center:** Wide variety of health and wellbeing services; https://wellness.caltech.edu/ Links to an external site.
- **Counseling Services:** Free for all students, regardless of insurance plan; http://counseling.caltech.eduLinks to an external site.
- **Occupational Therapy:** Individual sessions and consultations on building healthy habits and routines, time management, planning and organization, and more. Free for all students; http://ot.caltech.eduLinks to an external site.
- **Center for Inclusion and Diversity**: Resources concerning navigating diversity and inclusion, including staff who can speak with students about challenges of harassment and discrimination; http://diversitycenter.caltech.eduLinks to an external site.
- **Title IX**: Caltech's Title IX Coordinator (titleix@caltech.edu) works with students on issues related to sexual harassment, sexual misconduct, and sex discrimination; http://titleix.caltech.edu/ Links to an external site.
- **Caltech Accessibility Services for Students**: The Accessibility Services Specialist works with students with temporary medical conditions, or mental, physical or learning disabilities on accommodation requests and services; http://cass.caltech.eduLinks to an external site.
- **Residential Support**: Resident Associates (RAs) and Residential Life Coordinators (RLCs) are also resources for TAs and students; http://www.residentialexperience.caltech.edu/ Links to an external site.
- **Career Development Center:** Provides resources to help students make career decisions and implement career plans; http://www.career.caltech.eduLinks to an external site.